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Title: **SNS Control System Modeling-On the Peaking Problem of the Sensitivity Matrix *S***

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On the Peaking Problem of the Sensitivity Matrix S

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1. Background on the Feedback Control System Configuration

The sensitivity matrix is a defining tool of the transfer matrix (function) from the set point r to the error r-y.

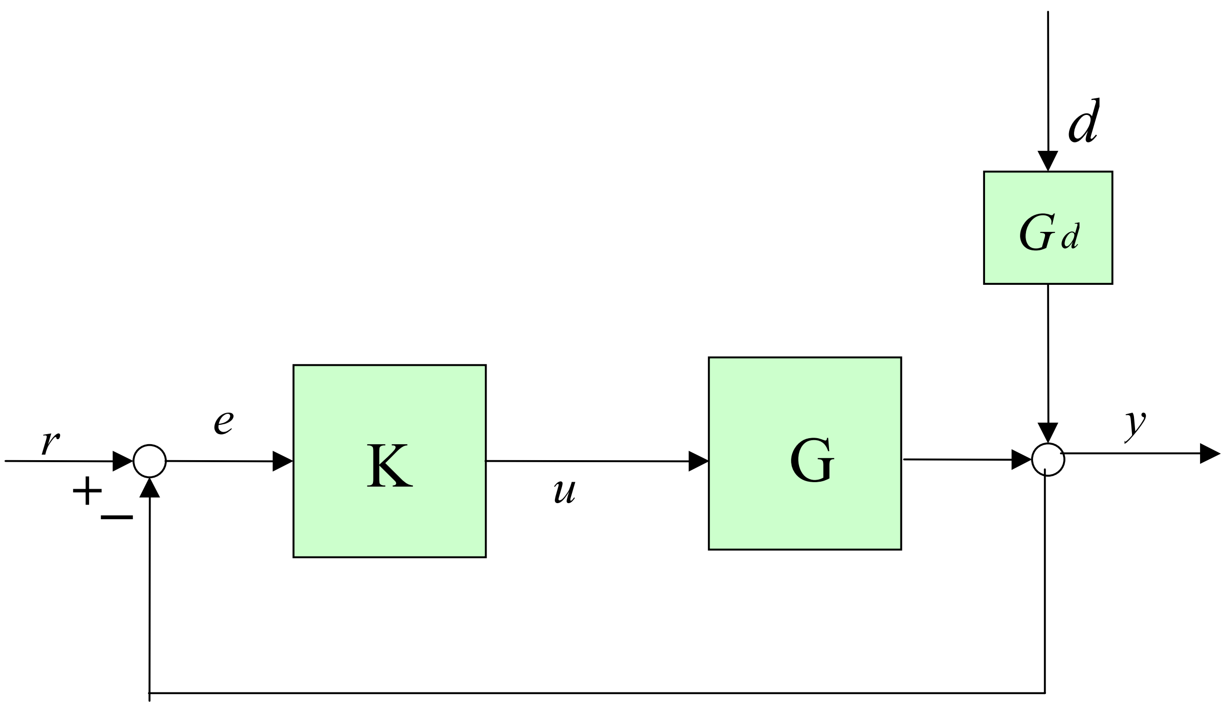

Figure 1 Feedback system configuration

The above figure shows the typical feedback control system configuration. The control purpose is to follow (track) the reference (set point) r, $\|r\|_2 \le 1$ and reject the effect of the scaled disturbance d, $\|d\|_2 \le 1$.

The error is given by

$$e = r - y = (I + GK)^{-1} r - (I + GK)^{-1} G_d d$$

and the output is given by

$$y = GK(r - y) = (I + GK)^{-1} GKr - (I + GK)^{-1} G_d d \; .$$

In control theory the sensitivity matrix is defined as

$$S = (I + GK)^{-1}$$

and the complementary sensitivity matrix is defined as

$$T = I + GK)^{-1} GK$$

and the loop transfer matrix is defined as

$$L = GK \; .$$

Roughly speaking, the sensitivity matrix is the transfer matrix from the set point to the error and the complementary sensitivity matrix represents the transfer matrix from the set point *r* to the output *y*.

The sensitivity matrix defines the transfer matrix from the reference (set point) *r* to the error *e* by

$$\underline{\sigma}(S(j\omega)) \le \frac{\|e\|_2}{\|r\|_2} \le \overline{\sigma}(S(j\omega))$$

where $\underline{\sigma}(S(j\omega))$ and $\overline{\sigma}(S(j\omega))$ are the minimal singular value and the maximal singular value of $S(j\omega)$. Usually, $\overline{\sigma}(S(j\omega))$ is defined as the infinite norm, $\|S(j\omega)\|_\infty$ of $S(j\omega)$. For a normalized reference, r, $\|r\|_2 \le 1$, the plot of $\overline{\sigma}(S(j\omega)) = \|S(j\omega)\|_\infty$ represents the error of the reference tracking. Also, the closed loop system bandwidth is defined as the frequency where $\|S(j\omega)\|_\infty$ crosses the –3dB point from the below.

2. The Peaking in the Sensitivity Matrix

In SNS, the normal conducting linac section consists of RFQ, DTL tanks, and CCL modules. The low level RF control system for a cavity is well matched with the feedback system configuration shown in figure 1. A cavity is driven by a high power RF amplifier, a klystron. The cascade of a klystron and a cavity is considered as the plant G in figure 1. A PI controller is used for the field stabilization. The PI feedback controller is considered as the controller K in figure 1. From the perspective of a cavity, the beam current is an exogenous disturbance. The beam current input is considered as the block G_d in figure 1. In addition, there are several sources of time delays: time delay in the waveguide, time delay in cavity field signal pickup cable, forward control signal loop delay, delay inside a klystron, and time delay due to the DSP computation. These time delays must be included in the block diagram of figure 1 at the appropriate locations, in order to obtain the more exact model of the low level RF control system.

Figure 2 shows the magnitude responses of the closed loop system of a DTL control system. In the magnitude response of the sensitivity matrix, the magnitude 10^{-2} means the 1.0 % error. In that figure, we observe the peaking of the sensitivity matrix plot at 4.5e5 rad/sec (71.6 kHz). Typically, peaking indicates a possibility of instability, particularly at high frequencies. When we consider the meaning of the sensitivity matrix, it seems that we are not worried about that peaking because the reference (set point) trajectory we use does not have that frequency component. If a reference has that frequency component, in order to achieve the set point tracking of the closed loop system, high closed loop system bandwidth is required. I mentioned that sensitivity matrix defines the transfer matrix from the reference to the error and figure 2 gives the equation.

If we are still nervous about that peaking, by reducing the PI feedback gain matrices, we decrease the peaking. However, this sacrifices the closed loop system bandwidth and degrades the disturbance attenuation performance of the closed loop system. Since usually the transfer function G_d is low pass filter-like, and since the decrease of the closed loop system bandwidth implies the shift of the plot of $\overline{\sigma}(S(j\omega))$ to the left in the frequency domain, the plot of the singular value $\overline{\sigma}((I+GK)^{-1}G_d)$ of transfer matrix $(I+GK)^{-1}G_d$ moves to the left in the frequency domain. Therefore, the frequency range of the disturbance d which can be attenuated decreases.

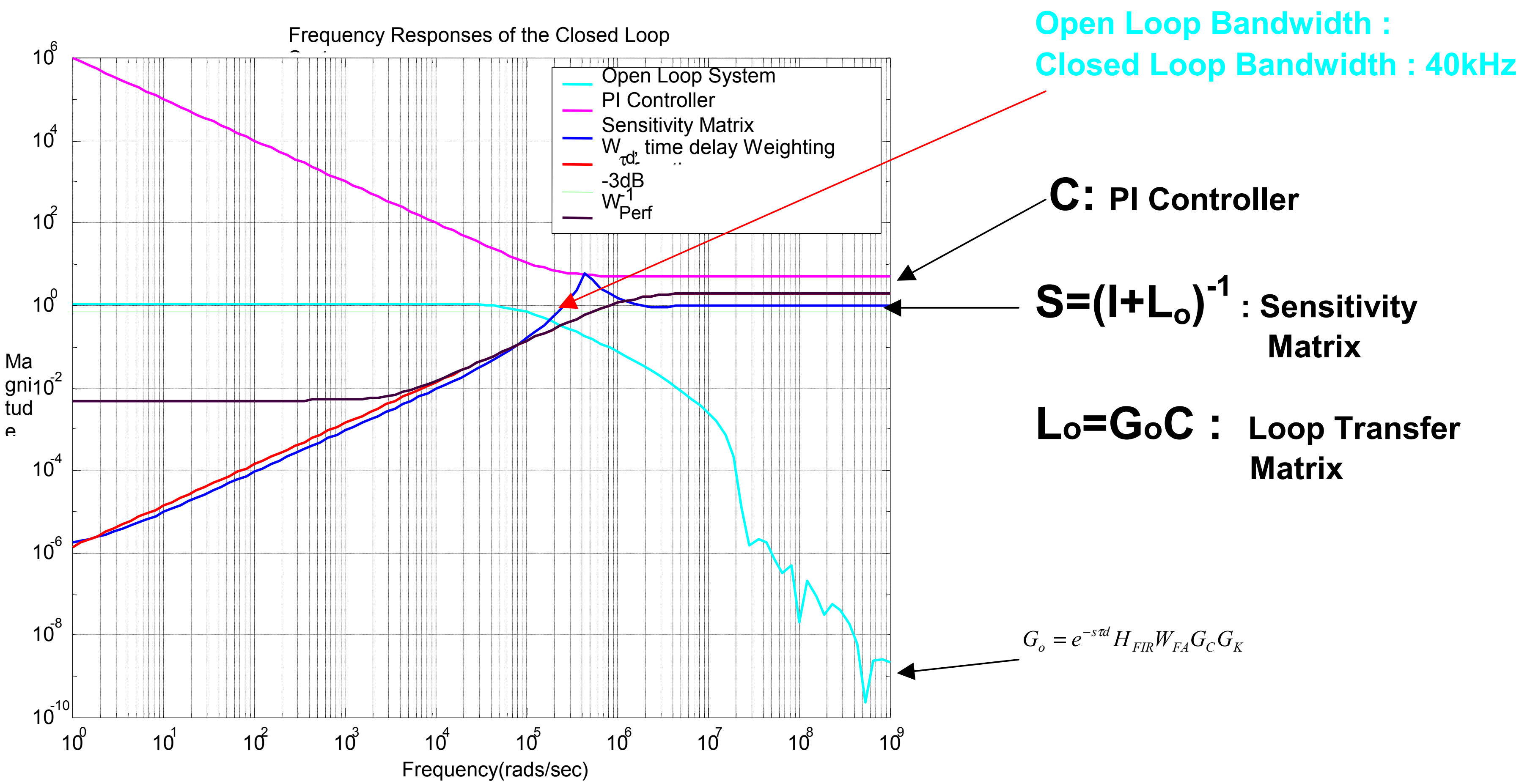

Figure 2 Magnitude Responses

Why is there peaking in the sensitivity matrix and complementary sensitivity matrix?

The first reason is the right half plane zeros due to the time delay. In the model, the time delay is approximated by Pade approximation. Since the time delay is an infinite-dimensional system, the Pade approximation yields right half plane zeros regardless of the order of the approximation.

The second reason is due to the FIR filter in the field control system. The FIR filter design based on the Remez exchange algorithm yields zeros near the unit circle in the z-plane with some inside the unit circle and others outside the unit circle. By the bilinear transformation, the discrete time FIR filter is transformed to the continuous time system in s-domain for frequency domain analysis. In that process, the zeros outside the unit circle in the z-plane are transformed to the zeros in the right half plane of the s-plane and the zeros inside the unit circle are transformed to the zeros in the left half plane of the s-plane. Also, the discrete time FIR filter has poles at the origin in the z-plane. When they are transformed to the s-plane, poles at the origin of s-plane result.

Example) In order to clarify this, an example is considered (figure 3). The open loop system is given by

$$g_o = \frac{10}{s+10}.$$

Also, in the loop, there is a time delay, $\tau d = 0.01\sec$. This time delay is implemented in Simulink as $e^{-s\tau d}$. Time delay is approximated by Pade Approximation which is given by

$$e^{-s\tau d} \approx \frac{\left(\frac{-\tau d}{2n}s+1\right)^n}{\left(\frac{\tau d}{2n}s+1\right)^n}$$

where n is the order of approximation. The approximation shows that time delay has n right-half plane zeros. As n increases, the number of right half plane zeros increases.

With time delay being included in the open loop system, the time delayed open loop system is

$$g_{od} = \frac{10}{s+10}e^{-s\tau d} \approx \frac{10}{s+10}\frac{\left(\frac{-\tau d}{2n}s+1\right)^n}{\left(\frac{\tau d}{2n}s+1\right)^n}.$$

Assume a feedback controller of proportional gain, K.

Then, sensitivity matrices (functions) are given by

$$S_o = (1+g_o K)^{-1} = \frac{s+10}{s+(10+10K)}$$

$$S_{od} = (1+g_{od}K)^{-1} = \frac{(s+10)\left(\frac{\tau d}{2n}s+1\right)^n}{(s+10)\left(\frac{\tau d}{2n}s+1\right)^n + 10\left(-\frac{\tau d}{2n}s+1\right)^n K}$$

where S_o and S_{od} represent the sensitivity matrices without time delay and with time delay, respectively.

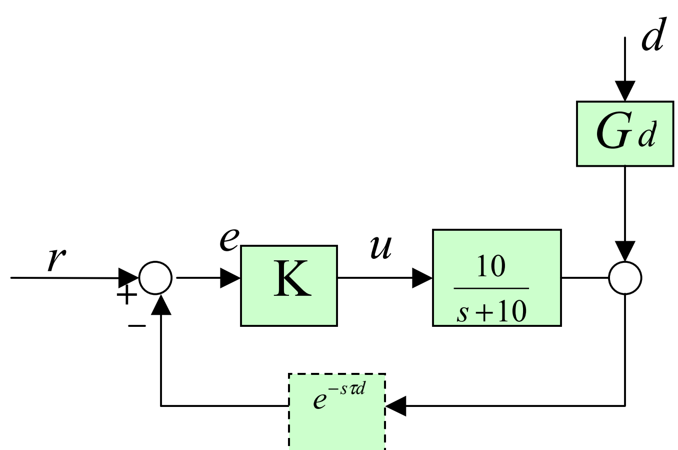

Figure 3 Feedback system configuration of example

Figure 4 shows the magnitude plots of sensitivity matrices (functions) when the gain K is 5.0 and the order of Pade approximation, *n,* is 10. The peaking is about 1.54.

The peaking is unavoidable but it can be reduced by reducing the proportional gain K. Figure 5 shows the cases where $K = 5.0$, $K = 2.5$, $K = 1.0$. Figure 5 shows that the peaking can be reduced by reducing the proportional gain. However, it yields the decrease of the closed loop system bandwidth which plays the main role in attenuating the external disturbance d. For $K = 5.0$, the closed loop system bandwidth is ~40 rad/sec, and for $K = 2.5$, the closed loop system bandwidth is ~25 rad/sec, and for $K = 1.0$, the closed loop system bandwidth is ~13 rad/sec.

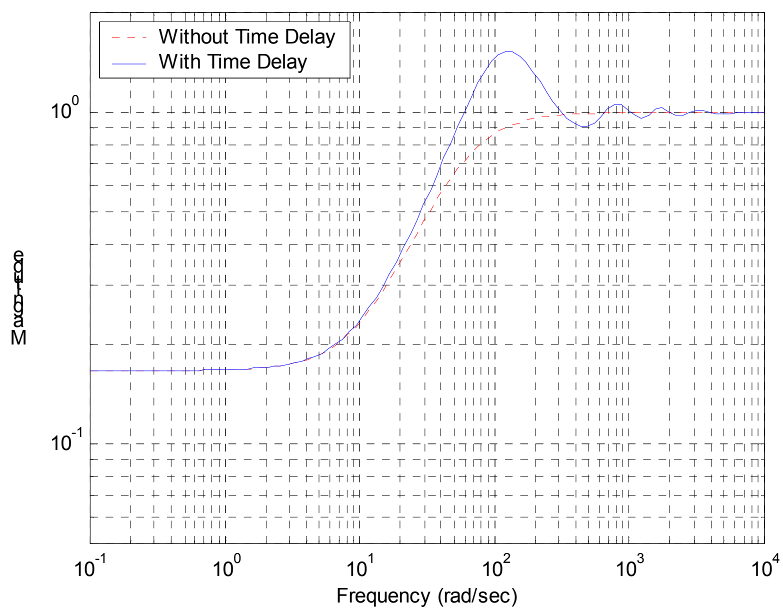

Figure 4 Magnitude of Sensitivity Matrix (function) when K=5.0, n=10

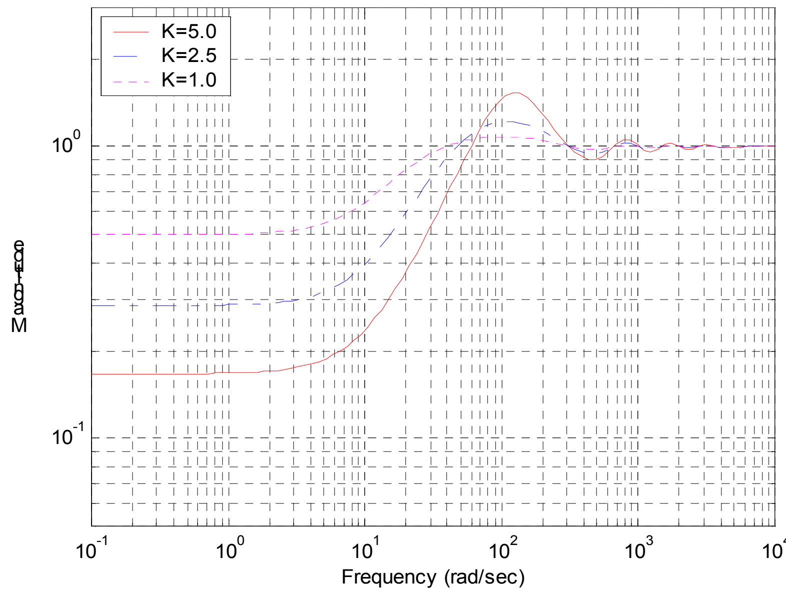

Figure 5 Magnitude of Sensitivity Matrices (functions) when $K = 5.0$, $K = 2.5$, $K = 1.0$, and n=10

3. Reduction of the peaking in our system

The plot shown in figure 2 is when the PI feedback gain matrices are at their critical values (maximal values) which are not violating the constraint due to time delay on the closed loop system bandwidth (inverse of the time delay in rad/sec),

$$\omega_{BW} \leq \omega_C < \frac{1}{\tau_d}$$

where ω_{BW} is the closed loop system bandwidth in rad/sec at which $\overline{\sigma}(S(j\omega))$ crosses –3 dB ($\frac{1}{\sqrt{2}}$) from the below and ω_C is the gain crossover frequency in rad/sec at which $\overline{\sigma}(L(j\omega))$ crosses 0 dB (1) from the above. For simulations, all conditions are kept the same.

Klystron delay: 150 $n\sec$

Waveguide delay: 116.5 $n\sec$ (100 ft)

Cable delay: 121 $n\sec$ (100 ft)

The total time delay is $1.392\,\mu\sec$.

The maximal PI feedback Gain Matrices based on the assumption of $1.392\ \mu\sec$ loop delay are

$$K_{P\max} = 5.0\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad K_{\mathrm{I}max} = 1.0e+006\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Proportional Gain Matrix	**Integrator Gain Matrix**	**Closed Loop System Bandwidth (kHz)**	**Sensitivity Matrix Peaking**	**Comp. Sensitivity Matrix Peaking**
$K_{P\max}$	$K_{\mathrm{I}max}$	40	6.3764	5.5985
$K_P = K_{P\max}$	$K_I = 0.5K_{\mathrm{I}max}$	36.6	3.892	3.2121
$K_P = 0.5K_{P\max}$	$K_I = 0.5K_{\mathrm{I}max}$	25.5	2.6109	2.3131
$K_P = 0.5K_{P\max}$	$K_I = 0.25K_{\mathrm{I}max}$	20.7	1.7833	1.2736
$K_P = 0.25K_{P\max}$	$K_I = 0.25K_{\mathrm{I}max}$	15.1	1.9644	1.8663
$K_P = 0.25K_{P\max}$	$K_I = 0.1K_{\mathrm{I}max}$	11.1	1.3084	1.2755
$K_P = 0.1K_{P\max}$	$K_I = 0.1K_{\mathrm{I}max}$	8.1	1.6709	1.4255

Open Loop System Bandwidth: 15.9 kHz